Logic-based Benders Decomposition and Binary Decision Diagram Based Approaches for Stochastic Distributed Operating Room Scheduling

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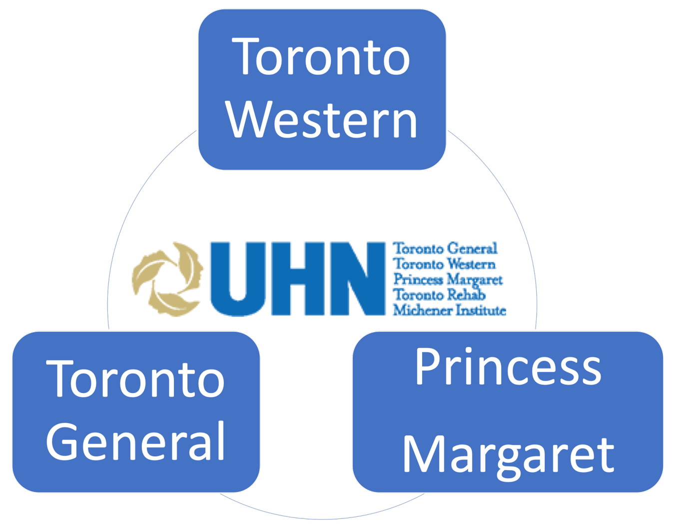
MODEL SETUP

The SDORS Problem



Time/Day	Mon	Tue	Wed	Thurs	Fri
9am-11am	Patient1	Patient1	Patient 1	Patient1	Patient1
11am-1pm		Patient2		Patient2	
1pm-3pm	Patient2				Patient2
3pm-5pm	Patient3		Cancelled	Patient3	Cancelled
Overtime					

SDORS: Stochastic Distributed Operating Room Scheduling

- Hospitals share operating rooms (ORs) and waiting lists
- Schedule patients to hospitals and ORs
- Surgery durations are stochastic
- Cancel a surgery if it is expected to finish after working hours

Model Novelty

★ Modeling the distributed OR scheduling problem in a stochastic setting

Goals

✓ Robust schedule
✓ Decrease costs
✓ Shorten waiting time
✓ Reduce cancellation

Decisions

u_{hd}: Open hospital
y_{hdr}: Open room
x_{hdpr}: Assign patient
w_p: Postpone patient
z_{hdpr}: Accept patient

The Model

1st stage: Schedule patients to (hospital, room, day) → observe durations → 2nd stage: Cancel patients if there is overtime

▶ 2-stage stochastic integer program (2SIP):

$$\min \quad f(u,y,x,w) + \mathbb{E}_{\mathbf{T}}[Q(\mathbf{x},\mathbf{y},\mathbf{T})]$$

$$\text{s.t.} \quad \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}_h} x_{hdpr} = 1 \quad \forall p \in \mathcal{P}'$$

$$\sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}_h} x_{hdpr} + w_p = 1 \quad \forall p \in \mathcal{P}\setminus\mathcal{P}'$$

$$y_{hdr} \le u_{hd} \quad \forall h\in\mathcal{H}; d\in\mathcal{D}; r\in\mathcal{R}_h$$

$$x_{hdpr} \le y_{hdr} \quad \forall h\in\mathcal{H}; d\in\mathcal{D}; p\in\mathcal{P}; r\in\mathcal{R}_h$$

$$y_{hdr} \le y_{hd,r-1} \quad \forall h\in\mathcal{H}; d\in\mathcal{D}; r\in\mathcal{R}_h\setminus\{1\}$$

$$u_{hd}, y_{hdr}, x_{hdpr}, w_p \in \{0,1\} \quad \forall h\in\mathcal{H}; d\in\mathcal{D}; p\in\mathcal{P}; r\in\mathcal{R}_h$$

▶ The cancellation cost $Q(\mathbf{x},\mathbf{y},\mathbf{T})$:

$$\min \quad \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}_h}\sum_{p\in\mathcal{P}} c_p^{\text{cancel}}(x_{hdp} - z_{hdpr})$$

$$\text{s.t.} \quad \sum_{p\in\mathcal{P}} T_p z_{hdpr} \le B_{hd} y_{hdr} \quad \forall h\in\mathcal{H}; d\in\mathcal{D}; r\in\mathcal{R}_h$$

$$z_{hdpr} \le x_{hdpr} \quad \forall h\in\mathcal{H}; d\in\mathcal{D}; p\in\mathcal{P}; r\in\mathcal{R}_h$$

$$z_{hdpr} \in \{0,1\} \quad \forall h\in\mathcal{H}; d\in\mathcal{D}; p\in\mathcal{P}; r\in\mathcal{R}_h$$

EXPERIMENTAL RESULTS

Vs. Commercial Solver

instance	Time / Gap			
(pp-h-d-r)	MIP	2-BDD	2-LBBD	3-LBBD
10-2-3-3	4%	3%	3%	17 (min)
25-2-3-3	13%	9%	12%	9%
10-3-5-5	6%	5%	6%	21 (min)
25-3-5-5	60%	17%	20%	-
50-3-5-5	61%	21%	21%	-
75-3-5-5	-	32%	21%	-

- Time limit: 30 minutes
- MIP: CPLEX 12.8
- - : gap > 100%

⇒ Decomposition outperforms MIP

⇒ For 2-stage decomposition, BDD-based cuts have a small advantage over LBBD cuts

⇒ 3-stage decomposition is very fast in small instances

Vs. Deterministic Model

instance	Cancel. Rate (%)		Utiliz. Rate (%)	
(pp-h-d-r)	Deter.	Sto.	Deter.	Sto.
10-3-5-5	16	1	3	5
25-3-5-5	19	2	9	11
50-3-5-5	16	2	18	23
75-3-5-5	15	5	29	33

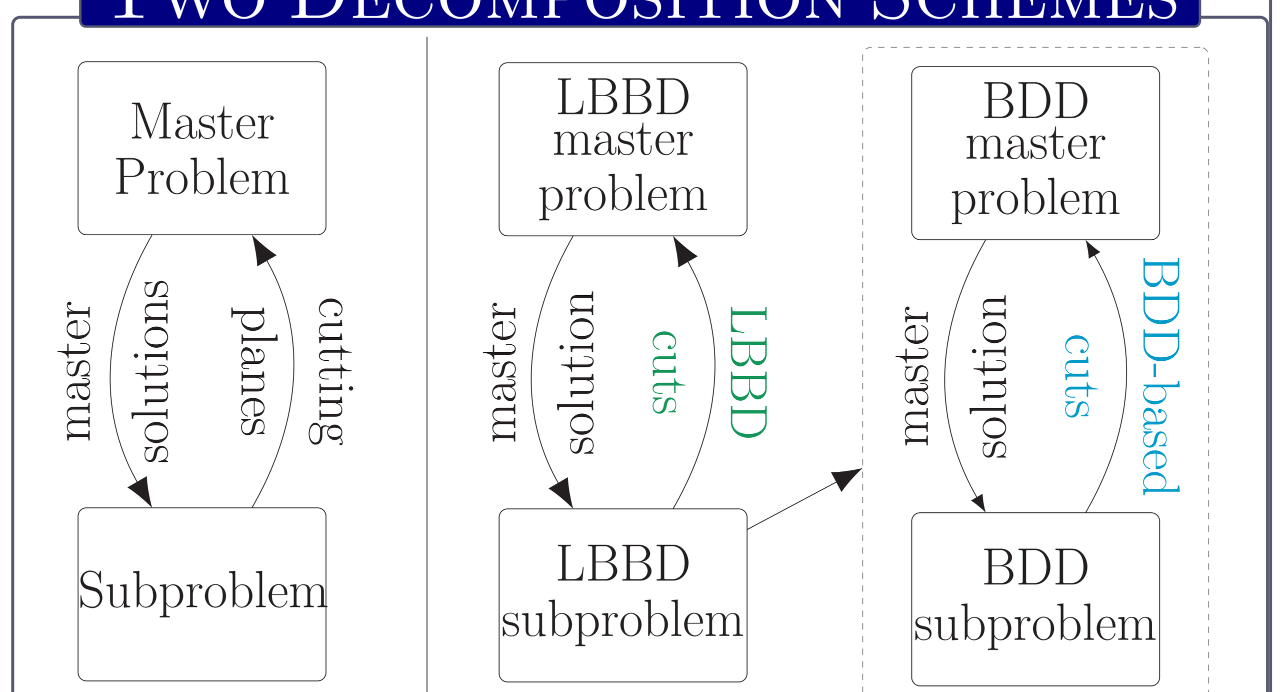
⇒ Stochastic model decreases cancellation and improves utilization

SOLUTION METHODS

Sample Average Approximation (SAA)

- Approximate 2SIP with SAA
- T_p^s: simulated surgery duration in scenario s
- z_{hdpr}^s: acceptance decision in scenario s

Two Decomposition Schemes



Algorithmic Contributions

★ **LBBD optimality cuts** for 2SIP
★ Adapted first fit decreasing (FFD) algorithm
★ Problem-specific subproblem relaxation
★ "Early stop" scheme

Two-stage Decomposition

▶ Master problem:

$$\min \quad f(u,y,x,w) + \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}}\sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} Q_{hdr}^s$$

s.t. assignment constraints on $x_{hdpr}, y_{hdr}, u_{hd}$
[BDD-based cuts or LBBD cuts]

▶ Subproblem:

$$\bar{Q}_{hdr}^s = \min \quad \sum_{p\in\mathcal{P}} c_p^{\text{cancel}}\left(\hat{x}_{hdpr} - z_{hdpr}^s\right)$$

s.t. assignment & duration constraints on z_{hdpr}^s

- Logic-Based Benders Decomposition (LBBD) cut:

$$Q_{hdr}^s \ge \bar{Q}_{hdr}^s - \sum_{p\in\hat{\mathcal{P}}_{hdr}} c_p^{\text{cancel}}\left(1 - x_{hdpr}\right)$$

- Binary Decision Diagram (BDD) based cut:

$$Q_{hdr}^s \ge \bar{\pi}_r + \sum_{p\in\mathcal{P}}\left(c_p - \left(\max_{a\in\mathcal{A}_{hdpr1}^s} \bar{\xi}_a\right)\right)x_{hdpr}$$

* $\bar{\xi}_a$, $\bar{\pi}_r$: dual optimal solutions of subproblem's BDD reformulation

- Classical Benders cuts from subproblem's linear programming (LP) relaxation

+ Fast for large instances

– Does not perform well in small instances

Three-stage Decomposition

▶ LBBD master problem:

$$\min \quad f(u,y,x,w) + \sum_{h\in\mathcal{H}}\sum_{d\in\mathcal{D}} Q_{hd}$$

s.t. assignment constraints on x_{hdp}, y_{hd}, u_{hd}
[LBBD cuts]

▶ LBBD subproblem (2SIP):

$$\bar{Q}_{hd} = \min \quad \frac{1}{|\mathcal{S}|}\sum_{s\in\mathcal{S}}\sum_{r\in\mathcal{R}_h}\sum_{p\in\mathcal{P}} c_p^{\text{cancel}}\left(x_{pr} - z_{pr}^s\right)$$

s.t. assignment & duration constraints on x_{pr}, z_{pr}^s

- LBBD cut:

$$y_{hd} \le \hat{y}_{hd} \Rightarrow Q_{hd} \ge \bar{Q}_{hd} - \sum_{p\in\hat{\mathcal{P}}_{hd}} \bar{Q}_{hd}\left(1 - x_{hdp}\right)$$

- Classical Benders cuts from subproblem LP relaxation
- Further decompose the LBBD subproblem with the BDD-based method

+ Fast for small instances

– Slow for large instances, due to difficult LBBD subproblems

Algorithmic Enhancements

♦ Adapted FFD heuristic
  – Adapt FFD method to get an initial solution
  – Derive additional constraints from this solution

♦ Subproblem relaxation
  – For the 2-stage decomposition:

$$Q_{hdr}^s \ge \left(\min_{p\in\mathcal{P}} \frac{c_p^{\text{cancel}}}{T_p^s}\right)\left(\sum_{p\in\mathcal{P}} T_p^s x_{hdpr} - B_{hd}\right)$$

  – Similar for 3-stage decomposition

♦ "Early stop" scheme
  – Avoid solving difficult subproblems
  – Stop solving the subproblem if *globalUB < incumbentOptCost*+$\bar{Q}_{hd}^{\text{LB}}$

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