Copositive Duality for Discrete Energy Markets

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Design A Pricing Scheme for Energy Markets with Discreteness

- Pricing is central to energy markets
- Electricity prices are based on shadow prices
  - Idealized market structure
- Discrete decisions in day-ahead market: start-up, on/off statuses
- Our solution: convexification of MIP

Outline

- Convexification of Unit Commitment using copositive programming
- Pricing Scheme in Discrete Energy Markets
    - Pricing and individual rationality in spot market
    - Pricing and individual rationality in day-ahead market
- Cutting plane algorithm for copositive programs

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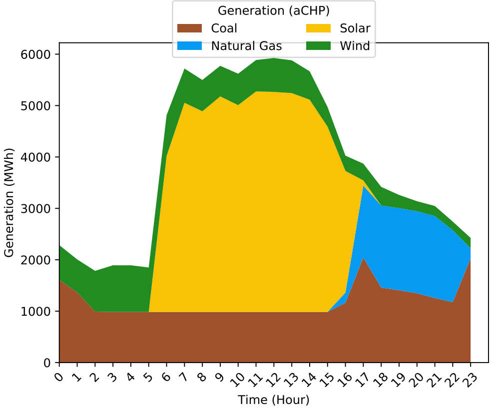
Unit Commitment (UC) Problem

- In the day-ahead market, decide the operation schedule of generators at each hour



MIP Model for Unit Commitment

$$
\begin{aligned}
\min \quad & \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \left(c_g^p p_{gt} + c_g^u u_{gt} \right) & \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} p_{gt} = d_t & \forall t \in \mathcal{T} \\
& \mathbf{a}_{jgt}^{\phi \top} \mathbf{x} = b_{jgt} & \forall j = 1, \ldots, m, g \in \mathcal{G}, t \in \mathcal{T} \\
& \mathbf{x} \in \mathbb{R}_+^n & \\
& z_{gt} \in \{0, 1\} & \forall g \in \mathcal{G}, t \in \mathcal{T}
\end{aligned}
$$

- p_{gt}: production level
- u_{gt}: turn on decision
- z_{gt}: on/off status
- $\ddot{\phi}$: slack variables
- $\mathbf{x}^\top = (\mathbf{z}^\top, \mathbf{u}^\top, \mathbf{p}^\top, \ddot{\phi}^\top)$

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MIP $\rightarrow$ Completely Positive Programming (CPP) (Burer, 2009)

$\mathcal{P}^{\text{MIP}}$ (nonconvex):

$$\begin{aligned} \min \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} \quad & a_j^\top \mathbf{x} = b_j, && \forall j = 1, ..., m \\ & x^k \in \{0, 1\}, && \forall k \in \mathcal{B} \\ & \mathbf{x} \in \mathbb{R}^n_+ \end{aligned}$$

$\mathcal{P}^{\text{CPP}}$ (convex):

$$\begin{aligned} \min \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} \quad & \mathbf{a}_j^\top \mathbf{x} = b_j && \forall j = 1, ..., m \\ & \mathbf{a}_j^\top X \mathbf{a}_j = b_j^2 && \forall j = 1, ..., m \\ & x^k = X_{kk} && \forall k \in \mathcal{B} \\ & \begin{bmatrix} 1 & \mathbf{x}^\top \\ \mathbf{x} & X \end{bmatrix} \in \mathcal{C}^* \end{aligned}$$

- If $x^k \in \{0, 1\}$, then $x^k = (x^k)^2$
- Let $X = \mathbf{x}\mathbf{x}^\top$, Enforce $x^k = X_{kk}$
- Constraints to enforce $X = \mathbf{x}\mathbf{x}^\top \rightarrow$ *there are different ways to do this for MIQP!*
  - Reformulation-Linearization Technique (RLT) constraint: $\mathbf{a}_j^\top X \mathbf{a}_j = b_j^2$
  - $\begin{bmatrix} 1 & \mathbf{x}^\top \\ \mathbf{x} & X \end{bmatrix} \in \mathcal{C}^*$
- Strong duality holds for CPP under regularity condition is satisfied.

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Setup of the Energy Market

- Supply: power plants, demand: utilities
- Independent system operator (ISO) holds auctions to match supply and demand
  - Day-ahead market: unit commitment
  - Spot market: no discrete decision

Pricing Scheme in Spot Market

- Spot market: ISO minimizes total cost

$$
\begin{aligned}
\min_{p_{gt}} \quad & \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} c_g^p p_{gt} \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}} p_{gt} = d_t, \quad \forall t \in \mathcal{T} \qquad (\lambda_t) \\
& (p_{gt}) \in X'_{gt}, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}
\end{aligned}
$$

- Let the optimal primal and dual solution be p_{gt}^* and λ_t^*.
- λ_t^* is the electricity price: More demand $\rightarrow$ more expensive technology $\rightarrow$ higher λ_t^*

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λ_t^* Guarantees Individual Rationality in Spot Markets

- Profit-maximizing problem for g has the same solution as the ISO's problem:

$$\max_{p_{gt}} \quad \sum_{t \in \mathcal{T}} (\lambda_t^* - c_g^p) p_{gt}$$
$$\text{s.t.} \quad (p_{gt}) \in X'_{gt}, \quad \forall t \in \mathcal{T}$$

- How to prove this? Decompose the Lagrangified ISO's problem

Proof for Individual Rationality in Spot Markets

- Lagrangify the demand constraint in the min-cost problem using λ_t^*. Due to convexity, p_{gt}^* is optimal to the following:

$$\min_{p_{gt}} \quad \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} c_g^p p_{gt} + \sum_{t\in\mathcal{T}} \lambda_t^* (\sum_{g\in\mathcal{G}} p_{gt} - d_t)$$

$$\text{s.t.} \quad (p_{gt}) \in X'_{gt}, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}$$

- Drop constant term $\lambda_t^* d_t$, reverse the sense:

$$\max_{p_{gt}} \quad \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} (\lambda_t^* - c_g^p) p_{gt}$$

$$\text{s.t.} \quad (p_{gt}) \in X'_{gt}, \quad \forall g \in \mathcal{G}, t \in \mathcal{T}$$

- Decomposable by g

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Pricing for Markets with Discrete Decisions is Challenging

- No dual price in MIP
- Literature on discrete energy market
  - Restricted pricing [O'Neil et al., 2005]
  - Convex hull pricing [Hogan and Ring, 2003; Gribik et al., 2007]
    Extended locational marginal pricing
- Literature on indivisible goods
  - Discrete convexity [Danilov et al., 2001; Baldwin and Klemperer, 2019]
  - Alpha-price mechanism [Milgrom and Watt, 2022]
- Still an open question

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Recap: Unit Commitment Problem & CPP Reformulation

$$
\begin{aligned}
\mathcal{UC} : \min \quad & \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} \left(c_g^p p_{gt} + c_g^u u_{gt} \right) & & \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}} p_{gt} = d_t & \forall t \in \mathcal{T} & \quad (\lambda_t) \\
& \mathbf{a}_{jgt}^{\phi\top} \mathbf{x} = b_{jgt} & \forall j = 1, ..., m, g \in \mathcal{G}, t \in \mathcal{T} & \\
& z_{gt} \in \{0,1\} & \forall g \in \mathcal{G}, t \in \mathcal{T} &
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}^{\text{CPP}} : \min \quad & \mathbf{c}^\top \mathbf{x} & \\
\text{s.t.} \quad & \mathbf{a}_j^\top \mathbf{x} = b_j & \forall j = 1, ..., m \\
& \mathbf{a}_j^\top X \mathbf{a}_j = b_j^2 & \forall j = 1, ..., m \\
& x^k = X_{kk} & \forall k \in \mathcal{B} \\
& \begin{bmatrix} 1 & \mathbf{x}^\top \\ \mathbf{x} & X \end{bmatrix} \in \mathcal{C}^* &
\end{aligned}
$$

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Convexification of UC

- CPP reformulation:

$$
\begin{aligned}
\mathcal{UC}^{\text{CPP}} = \min \quad & \sum_{g\in\mathcal{G}} \sum_{t\in\mathcal{T}} \left(c_g^p p_{gt} + c_g^u u_{gt} \right) \\
\text{s.t.} \quad & \sum_{g\in\mathcal{G}} p_{gt} = d_t && \forall t \in \mathcal{T} && (\lambda_t) \\
& \mathbf{a}_{jgt}^{\phi\top} \mathbf{x} = b_{jgt} && \forall j = 1, \ldots, m, g \in \mathcal{G}, t \in \mathcal{T} && (\phi_{jgt}) \\
& \text{Tr}(\mathbf{a}_t^{\lambda} \mathbf{a}_t^{\lambda\top} X) = d_t^2 && \forall t \in \mathcal{T} && (\Lambda_t) \\
& \text{Tr}(\mathbf{a}_{jgt}^{\phi} \mathbf{a}_{jgt}^{\phi\top} X) = b_{jgt}^2 && \forall j = 1, \ldots, m, g \in \mathcal{G}, t \in \mathcal{T} && (\Phi_{jgt}) \\
& z_{gt} = Z_{gt} && \forall g \in \mathcal{G}, t \in \mathcal{T} && (\delta_{gt}) \\
& \begin{bmatrix} 1 & x^\top \\ x & X \end{bmatrix} \in \mathcal{C}_{n+1}^* && && (\Omega)
\end{aligned}
$$

- Dual problem:

$$
\begin{aligned}
\mathcal{UC}^{\text{COP}} = \max \quad & \sum_{t\in\mathcal{T}} \left(d_t \lambda_t + d_t^2 \Lambda_t + \sum_{j=1}^{m} \sum_{g\in\mathcal{G}} \left(b_{jgt} \phi_{jgt} + b_{jgt}^2 \Phi_{jgt} \right) \right) \\
\text{s.t.} \quad & (\boldsymbol{\lambda}, \boldsymbol{\phi}, \boldsymbol{\Lambda}, \boldsymbol{\Phi}, \boldsymbol{\delta}, \Omega) \in \mathcal{F}^{\text{COP}}
\end{aligned}
$$

Shadow Price for Day-Ahead Market: Copositive Dual Pricing (CDP)

Let $(\boldsymbol{\lambda}^*, \boldsymbol{\phi}^*, \boldsymbol{\Lambda}^*, \boldsymbol{\phi}^*)$ be an optimal solution for $\mathcal{UC}^{\text{COP}}$. Under the CDP mechanism, at hour t the system operator:

(i) collects from the load:

$$d_t \lambda_t^* + d_t^2 \Lambda_t^* + \sum_{g \in \mathcal{G}} \sum_{j=1}^{m} \left(b_{jgt} \phi_{jgt}^* + b_{jgt}^2 \Phi_{jgt}^* \right)$$

(ii) pays to the generator g:

$$p_{gt}^* \lambda_t^* + P_{gt}^* \Lambda_t^* + \sum_{j=1}^{m} \left(\mathbf{a}_{jgt}^{\phi} \mathbf{x}^* \phi_{jgt}^* + \text{Tr}(\mathbf{a}_{jgt}^{\phi} \mathbf{a}_{jgt}^{\phi\top} X^*) \Phi_{jgt}^* \right) + \sum_{g' \in \mathcal{G} \setminus \{g\}} f(\Lambda_t^*, p_{gt}^*, p_{g't}^*)$$

Proof for Individual Rationality in Day-Ahead Markets

- Lagrangified CPP:

$$
\begin{aligned}
\min \quad & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\left(c_g^p p_{gt} + c_g^u u_{gt}\right) + \lambda_t^* \sum_{t\in\mathcal{T}}(d_t - \sum_{g\in\mathcal{G}} p_{gt}) + \Lambda_t^* \sum_{t\in\mathcal{T}}(d_t^2 - \mathrm{Tr}(\mathbf{a}_t^{\lambda}\mathbf{a}_t^{\lambda\top} X)) \\
\text{s.t.} \quad & \mathbf{a}_{jgt}^{\phi\top}\mathbf{x} = b_{jgt} && \forall j = 1,\dots,m, g\in\mathcal{G}, t\in\mathcal{T} \\
& \mathrm{Tr}(\mathbf{a}_{jgt}^{\phi}\mathbf{a}_{jgt}^{\phi\top} X) = b_{jgt}^2 && \forall j = 1,\dots,m, g\in\mathcal{G}, t\in\mathcal{T} \\
& z_{gt} = Z_{gt} && \forall g\in\mathcal{G}, t\in\mathcal{T} \\
& \begin{bmatrix} 1 & x^\top \\ x & X \end{bmatrix} \in \mathcal{C}_{n+1}^*
\end{aligned}
$$

- Idea: decompose this by g. But how?
  - First idea: make the conic constraint decomposable
  - Second idea: make $\Lambda_t^* = 0$
- A decomposable "Lagrangified MIP"

$$
\begin{aligned}
\min \quad & \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\left(c_g^p p_{gt} + c_g^u u_{gt}\right) + \lambda_t^* \sum_{t\in\mathcal{T}}(d_t - \sum_{g\in\mathcal{G}} p_{gt}) \\
\text{s.t.} \quad & \mathbf{a}_{jgt}^{\phi\top}\mathbf{x} = b_{jgt} && \forall j = 1,\dots,m, g\in\mathcal{G}, t\in\mathcal{T}
\end{aligned}
$$

Some Other Analytical Results

- System operators: Revenue from load = Payment to generators
- Generators: Total revenue = total costs (revenue neutrality)
- Supports market equilibrium
- A modified version of CDP that ensures individual revenue adequacy and uses linear prices
  - Results for CDP can be extended to this

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Solve the Dual Pricing Problem (A Copositive Program)

$$\mathcal{UC}^{\text{COP}} = \max \sum_{t\in\mathcal{T}} \left(d_t \lambda_t + d_t^2 \Lambda_t + \sum_{j=1}^{m} \sum_{g\in\mathcal{G}} \left(b_{jgt} \phi_{jgt} + b_{jgt}^2 \Phi_{jgt} \right) \right)$$
$$\text{s.t.} \quad (\boldsymbol{\lambda}, \boldsymbol{\phi}, \boldsymbol{\Lambda}, \boldsymbol{\Phi}, \boldsymbol{\delta}, \Omega) \in \mathcal{F}^{\text{COP}}$$

- $\mathcal{F}^{\text{COP}}$ includes conic constraint $\Omega \in \mathcal{C}_{n+1}$
- In literature: solved with SDP restriction
  - Define $\mathcal{S}^+$ and $\mathcal{N}$ $(\ni X_{ij} \geq 0, \forall i, j)$
  - $\mathcal{S}^+ + \mathcal{N} \subseteq \mathcal{C}$

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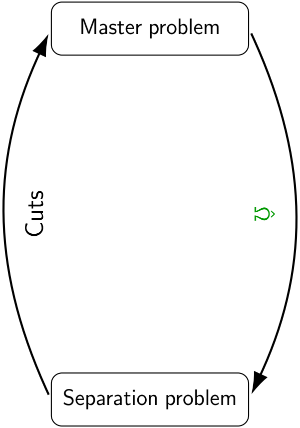
A Novel Cutting Plane Algorithm for Solving COP Exactly

$$
\begin{aligned}
\max_{\Omega, \boldsymbol{\lambda}} \quad & \mathbf{q}^\top \boldsymbol{\lambda} + \mathrm{Tr}(H^\top \Omega) \\
\text{s.t.} \quad & \mathbf{d}^\top \boldsymbol{\lambda} + \mathrm{Tr}(D_i^\top \Omega) = g_i, \quad \forall i = 1, \ldots, m \\
& \boldsymbol{\lambda} \geq \mathbf{0} \\
& \Omega \in \mathcal{C}^{n_c}
\end{aligned}
$$



- Separation problem [Anstreicher, 2020]:

$$
\begin{aligned}
\max_{w, \boldsymbol{u}, \boldsymbol{z}} \quad & w \\
\text{s.t.} \quad & \hat{\Omega}\mathbf{z} \leq -w\mathbf{1} + M(1 - \mathbf{u}) \\
& \mathbf{1}^\top \mathbf{u} \geq q \\
& w \geq 0 \\
& 0 \leq \mathbf{z} \leq \mathbf{u} \\
& \mathbf{u} \in \{0, 1\}^{n_c}
\end{aligned}
$$

- If $\bar{w} > 0$, add the cut: $\bar{\mathbf{z}}^\top \Omega \bar{\mathbf{z}} \geq 0$

Tighten the Master Problem Via Second-Order Cone Program

$$
\begin{aligned}
\max_{\Omega,\boldsymbol{\lambda}} \quad & \mathbf{q}^\top \boldsymbol{\lambda} + \mathrm{Tr}(H^\top \Omega) \\
\text{s.t.} \quad & \mathbf{d}^\top \boldsymbol{\lambda} + \mathrm{Tr}(D_i^\top \Omega) = g_i, && \forall i = 1, ..., m \\
& \boldsymbol{\lambda} \geq \mathbf{0} \\
& V + N = \Omega \\
& N \geq 0 \\
& V \in \mathcal{S}_n^+ \\
& V_{ii} \geq 0 && \forall i = 1, \cdots, n \\
& V_{ii} V_{jj} \geq V_{ij}^2 && \forall i \neq j; i, j = 1, \cdots, n \\
& \Omega \in \mathcal{C}^{n_c}
\end{aligned}
$$

- Converges to a feasible (not necessarily optimal) solution
- No worse than the SDP approximation $\mathcal{S}^+ + \mathcal{N} \subseteq \mathcal{C}$

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Comments and Performance of Cutting Plane Algorithms

- Straightforward to implement
  (vs simplicial partition [Bundfuss and Dür, 2008])
- Experiment on the max clique problem (2nd DIMACS dataset)
  - Cutting plane is more accurate and sometimes faster than the SDP approximation
- Significant speedup with the SOC-strengthened master problem
- To be improved:
  - Speed up the separation problem
  - Bounding the master problem at initialization
  - Tighter master problem
  - Other types of cuts

Summary

- A notion of duality for discrete problems
- Pricing scheme for discrete energy markets with good properties
- Novel cutting plane algorithm for copositive programs